EVALUATING THE SHORT-TERM EXCESS-RETURN: A NEW METHODOLOGY

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KEYWORDS	ABSTRACT
Mutual funds, performance measurement, short-term alpha.	In this paper, a new methodology for evaluating short-term excess return is suggested. The intuition behind this methodology is derived from the forward rate calculation and it does not require that the betas remain constant over time. The new methodology is compared with other short-term estimators and substantial score and ranking differences are found. In addition, the short-term estimators are analyzed based onaspects of expected value and variance and the conclusion is that the new methodology is the better one. Simulation tests support this result. Finally, the new methodology also yields performance scores and rankings that are the most consistent with their long-term counterparts.

1. INTRODUCTION

In this paper, a new short-term alpha estimation methodology is suggested: the differences method. The new methodology is analyzed and compared with other short-term alpha estimation methods and the conclusion is that the new methodology should be adopted.

The main idea in most of the classical measures of investment performance is to compare the return of a managed portfolio over some evaluation period to the return of a benchmark portfolio. Early literature develops measures of portfolio performance that continue to be used in much of the performance literature.

The Capital Asset Pricing Model (Sharpe (1964)) implies that all investors should hold a broadly diversified portfolio - the market portfolio - and safe assets in a portion according to their tastes for risk. Jensen (1968) uses the intercept of the factor model regressions to measure abnormal returns generated from picking stocks that outperform a risk-adjusted benchmark[1]. Other measures of portfolio performance developed by the early literature on portfolio performance are the Sharpe ratio (see Sharpe (1966)) and Treynor's measure (see Treynor (1965)).

Following the CAPM, the Arbitrage Pricing model of Ross (1976) allows for several risk factors to determine assets' expected returns, but leaves it up to empirical research to identify the risk factors. Fama and French (1996) and Carhart (1997) evaluate performance using three-factor or four-factor models derived from empirically observed patterns in stock portfolio returns.

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[1] In a recent paper Chance (2011) has suggested that traditional methods of measuring alpha may lead to positive bias.

Performance can be estimated over long-term as well as short-term horizons. In order to evaluate short-term excess return (short-term alpha) correctly, the performance evaluating method should be valid when implemented over a short period such as one-year or one-month. The common short-term excess-return estimation method researchers use is the out-of-sample alpha method. The method estimates alpha via an APT model that deviates from the standard APT model by using the betas from the preceding period.

Brennan, Chordia, and Subrahmanyam (1998) examine the relation among stock returns, measures of risk, and several non-risk security characteristics. They use the out-of-sample alpha to test the null hypothesis that expected returns depend only on the risk characteristics of the returns: that the loadings on Connor and Korajczyk (1988) or Fama and French (1993) factors are equal to zero. Chordia, Subrahmanyam, and Anshuman (2001) use the out-of-sample alpha to analyze the relation between expected equity returns and the level of trading activity as well as expected equity returns and the volatility of trading activity. Spiegel and Wang (2006) use the out-of-sample alpha to determine the degree to which idiosyncratic risk, liquidity, size, lagged returns and dollar volume explain cross sectional variation in stock returns. Chordia and Shivakumar (2006) use the out-of-sample alpha and find that a portfolio that is long in stocks with the highest earnings surprises and short in stocks with the lowest earnings surprises provides short-term excess return. Chordia, Huh and Subrahmanyam (2009) approach liquidity estimation from a theoretical perspective and recognize the analytic dependence of illiquidity on trading activity and information asymmetry. Then, they use the out-of-sample alpha methodology to compute risk-adjusted monthly returns and conclude that theory-based estimates of illiquidity are priced in the cross-section of expected stock returns. Ben-Rephael, Kadan and Wohl (2010) examine the profitability of buying illiquid stocks. They construct portfolios double sorted on both size and liquidity and then, for each size tercile and each month, they construct long-short liquidity-based portfolios. To evaluate the profitability of these portfolios, they estimate out-of-sample alphas relative to the four Fama-French factors[2].

Short-term alphas might also be important because of the momentum phenomenon (Jegadeesh and Titman (1993)) that predicts short-term persistence. A substantial body of research examines the persistence of mutual fund returns. Persistence is a crucial issue for investors who wish to find high-return funds: Can a fund that performed relatively well in the past be expected to do so again in the future? Persistence is also a crucial issue for fund managers since funds whose past returns are relatively high tend to attract relatively more new investment money.[3] This strand of research documents mutual fund return predictability over a longer horizon of five to ten years and attributes it to managers' exposure to different information or stock-picking talent. Over shorter-term horizons of one to three years, there is evidence of persistence, or momentum, in mutual fund performance that is attributed to "hot hands" or common investment strategies.[4] Much of this continuation seems to be explained by funds' holdings of momentum stocks (e.g., Carhart (1997),

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[2] See also Brennan, Chordia and Subrahmanyam (2009).

[3] For example, see Ippolito (1989), Brown, Harlow and Starks (1996), Gruber (1996), Chevalier and Ellison (1997), Sirri and Tufano (1998), Zheng (1999), Busse (2001), Goriaev, Nijman and Werker (2005).

[4] Persistence and momentum have been noted by many authors: Sharpe (1966), Treynor(1965), Carlson (1970), Ippolito (1992), Grinblatt and Titman (1992), Brown, Goetzmann, Ibbotson and Ross (1992), Goetzmann and Ibbotson (1994), Shukla and Trzcinka (1994), Hendricks, Patel and Zeckhauser (1993, 1997), Brown and Goetzmann (1995), Malkiel (1995), Gruber (1996), Elton, Gruber and Blake (1996), Carhart (1997), Graham and Harvey (1997), Otten and Bams (2002), Berk and Green (2004), Wermers (2004), Bollen and Busse (2004), Kaplan and Schoar (2005), Kosowski, Timmermann, Wermers and White (2006), Cremers and Petajisto (2009).

Grinblatt, Titman and Wermers (1995)).[5] Although momentum appears to be a widespread phenomenon, it may be due to superior management rather than to statistical properties of asset prices. Chen, Jegadeesh and Wermers (2000), for example, find weak evidence that funds with the best past performance have better stock-picking skills than funds with the worst past performance. Hunter, Kandel, Kandel and Wermers (2009) note that better estimation of alpha by sophisticated estimation methods may actually imply lower predictability of future returns, if these superior abilities are non-existent. On the other hand, if superior abilities do exist, sophisticated methods for the estimation of alpha should improve the predictability.

Short-term alpha is also important for practitioners since a significant element of their compensation is often tied to short-term performance.

Researchers show empirically that the systematic risk of stocks varies substantially over time (see, for example, Ferson and Harvey (1999)). In such a dynamic world it is unlikely that exposure to risk and style factors remains constant over time. Kumar (2009) finds that individual investors exhibit time-varying style preferences. Hence, fluctuations in factor exposures should be taken into account when measuring performance (Ferson and Schadt (1996), Bauer, Cosemans and Eichholtz (2007)).

The intuition behind the new short-term alpha estimation method suggested in this paper - the differences method - is derived from the forward rate calculation: it estimates the alpha of period n by dividing n-period performance results by the coinciding (n-1)-period performance results. The differences method, as opposed to the widely used out-of-sample alpha method, does not need to assume that the betas are constant over a short period of time.

The differences method is analyzed and compared with the out-of-sample alpha method and with the one-year regression method based on aspects of expected value and variance. The analysis implies that all three methods' estimators have the same expected value, and it demonstrates that the differences method's estimator has the lowest variance. Thus, the differences method is the better short-term alpha estimation method of the three methods examined in this paper. In addition, a simulationalsosupports this result. The simulation suggests that the differences method better estimates the short-term alpha since its estimator yields, on average, the smallest difference (whether with or without absolute value) between itself and the simulated alpha.

The three short-term alpha estimators are compared and substantial score and ranking differences are found between the short-term alpha estimates, though the correlations between the estimates are high. In addition, as the regression model gets more complicated, the correlation between the short-term alpha estimates decreases and the classification and ranking differences increase.

Finally, the short-term alpha estimators are compared to the standard alpha as extracted from a regression run over four-year and three-year periods (long-term alphas). The comparison shows that the differences method yields performance scores and rankings that are the most consistent with their long-term counterparts. The conclusion is that the differences method, as suggested in this paper, should be adopted.

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[5] At the same time, there is also contrary evidence of performance reversion: good past performance is not followed by good subsequent performance.

2. DERIVING ALPHA: THREE METHODS

This section specifies the three short-term alpha estimation methods examined in this paper.[6] Then, the short-term alpha estimation methods are analyzed and compared based on aspects of expected value and variance. The analysis implies that all three methods' estimators have the same expected value and the differences method's estimator has the lowest variance of all.

Given a four-year period (years 1 to 4), the estimation of the alpha of the fourth year (year 4) is of interest . Method 1, the one-year regression method, estimates the fourth-year alpha by running a regression over data from one-year (year 4) of security P's excess return on the relevant N benchmarks' excess return:

$$\widetilde{R}_{P,t,year=4} - \widetilde{R}_{f,t,year=4} = \sum_{i=1}^{N} \beta_{i,year=4} * (\widetilde{R}_{i,t,year=4} - \widetilde{R}_{f,t,year=4})$$

$$+ \alpha_{year=4} + \widetilde{\varepsilon}_{t,year=4}. \tag{1}$$

The regression is run using monthly data such that there are twelve return observations. Then the alpha is extracted from that regression, it is called $\alpha_{(method=1)}^{year=4}$, and it is used as a measure for security P's excess return over year 4.

Method 2 estimates the fourth year alpha by applying the widely used out-of-sample alpha method. For simplicity, the methodology is demonstrated via a one-benchmark regression model. First, a regression of security P's excess return on the benchmark's excess return is run over the preceding three-year period (years 1 to 3):

$$\widetilde{R}_{P,t,year=1-3} - \widetilde{R}_{f,t,year=1-3} = \beta_{year=1-3} * (\widetilde{R}_{B,t,year=1-3} - \widetilde{R}_{f,t,year=1-3})$$

$$+ \alpha_{year=1-3} + \widetilde{\varepsilon}_{t,year=1-3}. \tag{2}$$

Let beta as extracted from the regression above be $\beta_{year=1-3}$. Then, the alpha of year 4 is estimated via the equation:

$$\overline{R}_{P,year=4} - \overline{R}_{f,year=4} = \alpha_{year=4} + \beta_{year=1-3} * (\overline{R}_{year=4} - \overline{R}_{f,year=4}). \tag{3}$$

Let the out-of-sample alpha method's year 4 estimator be $\alpha_{(method=2)}^{year=4}$.

Third, a new methodology, the differences method (method 3), is suggested for evaluating the fourth-year alpha: calculate the four-year alpha ($\alpha_{year=1-4}$, estimated by a regression over years 1

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[6]The data is not assumed to be stationary. Thus, performance is examined over short time periods such as three and four years. However, all methodologies could be implemented over longer as well as shorter time periods.

to 4) and the three-year alpha ($\alpha_{year=1-3}$, estimated by a regression over years 1 to 3), and then estimate the fourth year (year 4) alpha, $\alpha_{(method=3)}^{year=4}$, as:

$$(1+\alpha_{(method=3)}^{year=4})^{12} = \frac{(1+\alpha_{year=1-4})^{48}}{(1+\alpha_{year=1-3})^{36}}.$$

(4)

Thus,

$$\alpha_{(method=3)}^{year=4} = \frac{(1+\alpha_{year=1-4})^{4}}{(1+\alpha_{year=1-3})^{3}} - 1.$$

(5)

Note that the out-of-sample alpha method, as well as the differences method, allow one to estimate alpha over shorter periods of time. For example, consider a 37-month period (months 1 to 37) and assume that the estimation of alpha for month 37 is wanted.

The out-of-sample alpha method can be used for evaluating the month-37 alpha. First, a regression of security P's excess return on the benchmark's excess return is run over the previous three-year period (years 1 to 3 or months 1 to 36):

$$\widetilde{R}_{P,t,year=1-3} - \widetilde{R}_{f,t,year=1-3} = \beta_{year=1-3} * (\widetilde{R}_{B,t,year=1-3} - \widetilde{R}_{f,t,year=1-3})$$

$$+ \alpha_{year=1-3} + \widetilde{\varepsilon}_{t,year=1-3}.$$

(6)

Let beta as extracted from the regression above be $\beta_{year=1-3}$. Then, the month-37 alpha is estimated via:

$$R_{P,month=37} - R_{f,month=37} = \alpha_{month=37} + \beta_{year=1-3} * (R_{month=37} - R_{f,month=37}).$$

(7)

The differences method can also be used for evaluating the month-37 alpha. The three-year (36-month) alpha ($\alpha_{year=1-3}$, estimated by a regression over years 1 to 3) and the 37-month alpha for months 1 to 37 ($\alpha_{month=1-37}$, estimated by a regression over months 1 to 37) are calculated, and then the month-37 alpha is estimated as:

$$\alpha_{(method=3)}^{month=37} = \frac{(1+\alpha_{month=1-37})^{37}}{(1+\alpha_{year=1-3})^{36}} - 1.$$

(8)

2.1 Expected Value and Variance of the Short-Term Excess-Return Estimators

In this subsection, the short-term excess return estimators described above are analyzed and compared using econometric tools. The analysis deviates from the standard regression framework and assumes that the Data Generating Process (DGP) is such that alpha and beta vary with time.

First, the new DGP's framework is analyzed. Then, the expected value and variance of the short-term alpha estimators are evaluated. The analysis results suggest that all estimators have the same expected value and the differences method estimator has the lowest variance.

Below are the framework's assumptions. For simplicity a one-benchmark framework is considerd.

Let the security return at time t be y_t. Let the benchmark return at time t be x_t, where x_t is constant at time t. Let beta at time t be β_t, where beta at time t is distributed $\beta_t \sim (\beta, \sigma_\beta^2)$. Let alpha at time t be α_t, where alpha at time t is distributed $\alpha_t \sim (\alpha, \sigma_\alpha^2)$. The framework assumes that there might be some persistence in the security's performance such that $COV(\alpha_t, \alpha_{t-1}) \neq 0$. It is also assumed that beta might reflect an investment strategy and thus it cannot vary much from the current period to the following one, such that $COV(\beta_t, \beta_{t-1}) \neq 0$. In addition, it is assumed that the investment strategy is influenced by its benchmark performance such that $COV(x_{t-1}, \beta_t) \neq 0$.

Assuming alpha and beta vary each period, the DGP is $y_t = \alpha_t + \beta_t x_t + u_t$, with $E(u_t) = 0$. The regression equation, on the other hand, includes alpha and beta estimators that do not vary with time: $y_t = \hat{\alpha} + \hat{\beta} x_t + \varepsilon_t$. The expected value of the regression equation is $\bar{y} = \hat{\alpha} + \hat{\beta}\bar{x}$.

The estimators for $\hat{\alpha}$ and $\hat{\beta}$ are $\hat{\alpha} = \bar{y} - \hat{\beta}\bar{x}$ and $\hat{\beta} = \dfrac{\sum(x_t - \bar{x})(y_t - \bar{y})}{\sum(x_t - \bar{x})^2}$, respectively, where $\bar{y} = \sum \dfrac{\alpha_t}{n} + \sum \dfrac{\beta_t x_t}{n} + \sum \dfrac{u_t}{n}$.

The following two results summarize the new DGP's framework.[7]

Result 1: The expected value of beta's estimator is $E(\hat{\beta}) = \beta$.

Result 2: The expected value of alpha's estimator is $E(\hat{\alpha}) = \alpha$.

Next, the expected value and variance of the three short-term alpha estimators are evaluated.[8]

First, consider the one-year regression method. Recall that the estimation of the alpha over the fourth year (year 4) is of interest. Let the alpha evaluated by the one-year regression method be $\alpha_{(1)}^4$, and $\alpha_{(1)}^4$ is extracted from the regression run over the 4th year data. Relating $\alpha_{(1)}^4$ to the

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[7] All proofs are provided in the Appendix.

[8] All proofs are provided in the Appendix.

previous discussion, $\alpha_{(1)}^4$ is practically $\hat{\alpha}$ of the fourth year, $\hat{\alpha}^4$.[9]Thus, $E(\alpha_{(1)}^4) = E(\hat{\alpha}^4) = \alpha$.
Let the variance of $\alpha_{(1)}^4$ be $VAR(\alpha_{(1)}^4) = VAR(\hat{\alpha}^4)$.

Second, consider the out-of-sample alpha method. Let the alpha evaluated by the out-of-sample alpha method be $\alpha_{(2)}^4$, and $\alpha_{(2)}^4$ is evaluated as follows: $\alpha_{(2)}^4 = \bar{y}^4 - \hat{\beta}^{1-3}\bar{x}^4$, where $\bar{y}^4$ is the fourth-year average security return, $\hat{\beta}^{1-3}$ is the beta extracted from a regression of the security excess return over the benchmark excess return, run over years 1 to 3, and $\bar{x}^4$ is the fourth-year average benchmark return.

Result 3: The expected value of $\alpha_{(2)}^4$ is $E(\alpha_{(2)}^4) = \alpha$ and its variance is $VAR(\alpha_{(2)}^4) = VAR(\hat{\alpha}^4) + (\bar{x}^4)^2(VAR(\hat{\beta}^4 - \hat{\beta}^{1-3}))$.

Third, consider the differences method. Let the alpha evaluated by the differences method be $\alpha_{(3)}^4$, and evaluate $\alpha_{(3)}^4$ as: $\alpha_{(3)}^4 = \dfrac{(1+\hat{\alpha}^{1-4})^4}{(1+\hat{\alpha}^{1-3})^3} - 1$, where $\hat{\alpha}^{1-3}$ is the alpha extracted from a regression of the security excess return over the benchmark excess return run over years 1 to 3, and $\hat{\alpha}^{1-4}$ is the alpha extracted from a regression of the security excess return over the benchmark excess return run over years 1 to 4.

Result 4: The expected value of $\alpha_{(3)}^4$ is $E(\alpha_{(3)}^4) \approx \alpha$ and its variance is $VAR(\alpha_{(3)}^4) \approx VAR(\hat{\alpha}^{1-4})$ or $\approx VAR(\hat{\alpha}^{1-3})$.

The discussion of the expected value and variance of the three short-term alpha estimators is summarized in table 1.

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[9] In the same manner, in the upcoming discussion, alpha estimated over year t to year t' is denoted as $\hat{\alpha}^{t-t'}$, beta estimated over year t to year t' is denoted as $\hat{\beta}^{t-t'}$, the average security return between year t and year t' is denoted as $\bar{y}^{t-t'}$ and the average benchmark return between year t and year t' is denoted as $\bar{x}^{t-t'}$.

Table 1: Analysisof The Short-Term Alpha Estimation Methods

	METHOD		
	ONE-YEAR REGRESSION	OUT-OF-SAMPLE ALPHA	DIFFERENCES
EXPECTED VALUE	α	α	α
VARIANCE	$VAR(\hat{\alpha}^4)$	$VAR(\hat{\alpha}^4) + (\bar{x}^4)^2 VAR(\hat{\beta}^4 - \hat{\beta}^{1-3})$	$VAR(\hat{\alpha}^{1-4})$ or $VAR(\hat{\alpha}^{1-3})$

Note that as the regression is run over a longer period (three or four years as opposed to one year), the alpha estimator's volatility decreases. So, it can be assumed that $VAR(\hat{\alpha}^{1-4}) \approx VAR(\hat{\alpha}^{1-3}) < VAR(\hat{\alpha}^4$). Given that the expected value of all estimators equals α and the differences method estimator has the lowest variance of all, the differences method's estimator, $\alpha^4_{(3)}$, is the best estimator for short-term excess return.

3. METHODOLOGY AND DATA

This section compares the short-term alpha estimators and it also compares the short-term alpha estimators to the standard alphas as extracted from a regression run over four-year and three-year periods (long-term alphas).

3.1 Data

Data is collected on several groups of funds using the Lipper classification of funds, as provided in CRSP (The Center for Research in Security Prices). The Lipper classification divides the world of non-specialized open-end equity funds into 12 groups based on the funds' style: LCCE (Large-Cap Core Funds), LCGE (Large-Cap Growth Funds), LCVE (Large-Cap Value Funds), MCCE (Mid-Cap Core Funds), MCGE (Mid-Cap Growth Funds), MCVE (Mid-Cap Value Funds), SCCE (Small-Cap Core Funds), SCGE (Small-Cap Growth Funds), SCVE (Small-Cap Value Funds), MLCE (Multi-Cap Core Funds), MLGE (Multi-Cap Growth Funds), and MLVE (Multi-Cap Value Funds). Funds' Lipper classifications are given and can change from one year to another. While analyzing the funds, only funds whose classification does not change over the entire period are included and analyzed.

Mutual fund data are collected from the CRSP database. The Fama-French factors and the additional momentum factor returns are taken from French's web site.[10] The three Fama-French factors are: 1) the performance of small stocks relative to big stocks (SMB, Small Minus Big), 2) the performance of value stocks relative to growth stocks (HML, High Minus Low), based upon the Fama-French Portfolios, and 3) the excess market returns, based on the value-weight return on all NYSE, AMEX, and NASDAQ stocks (from CRSP), and the one-month Treasury bill rate (from Ibbotson Associates). The momentum factor of Carhart (1997) is the average return on the two high prior return portfolios minus the average return on the two low prior return portfolios.

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[10] http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html.

3.1.1 Data Characteristics

The data contains ten years (2001-2010) of funds' returns. Table 2 reports the 2001-2010 average data characteristics.

Table 2: Data Summary - 2001-2010 Data Characteristics

LIPPER CLASSIFICATION	OBSERVATIONS	AVERAGE			WEIGHTED (BY TNA)	
		TOTAL NET ASSETS	EXPENSE RATIO	MANAGEMENT FEE	EXPENSE RATIO	MANAGEMENT FEE
LCCE	288.7	897.3	1.2%	0.6%	1.1%	0.6%
LCGE	206.0	860.2	1.3%	0.7%	1.2%	0.7%
LCVE	119.5	1,183.2	1.2%	0.6%	1.1%	0.6%
MCCE	118.1	647.0	1.4%	0.8%	1.3%	0.8%
MCGE	149.2	383.5	1.5%	0.8%	1.4%	0.8%
MCVE	75.2	791.9	1.3%	0.8%	1.2%	0.8%
MLCE	229.9	718.1	1.2%	0.6%	1.2%	0.6%
MLGE	146.1	1,132.8	1.5%	0.8%	1.4%	0.8%
MLVE	147.1	750.5	1.2%	0.7%	1.1%	0.7%
SCCE	208.3	399.9	1.3%	0.8%	1.2%	0.8%
SCGE	164.4	275.5	1.5%	0.9%	1.4%	0.9%
SCVE	87.5	343.5	1.4%	0.8%	1.3%	0.9%
SUM:	1940					
AVERAGE:		698.6	1.3%	0.7%	1.2%	0.7%

The average number of funds per year is 1940, of which 289 are classified as LCCE, 206 are classified as LCGE, 120 are classified as LCVE, 118 are classified as MCCE, 149 are classified as MCGE, 75 are classified as MCVE, 230 are classified as MLCE, 146 are classified as MLGE, 147 are classified as MLVE, 208 are classified as SCCE, 164 are classified as SCGE, and 88 are classified as SCVE. The 2001-2010 average Total Net Assets (TNA) of all funds is $698.6 million. The average expense ratio (management fee) charged by fund managers is 1.3% (0.7%) and the weighted (by TNA) average expense ratio (management fee) is 1.2% (0.7%). The yearly data characteristics show no significant deviations from the average data characteristics presented at Table 2.[11]

3.2 Methodology

This subsection describes the methodology for comparing the short-term alpha estimators as well as the methodology for comparing the short-term alpha estimators to the standard alphas as extracted from a regression run over three-year and four-year periods (long-term alphas).

First, the three short-term alpha estimation methods described above are applied to seven one-year periods (2004-2010) in order to evaluate alpha throughout each of the seven years. Thus, for fund

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[11]This data is not included in the body of the work, but it is available by request.

P and for method i=1,2,3, $\alpha_{(method=i)}^{2004}$, $\alpha_{(method=i)}^{2005}$, $\alpha_{(method=i)}^{2006}$, $\alpha_{(method=i)}^{2007}$, $\alpha_{(method=i)}^{2008}$, $\alpha_{(method=i)}^{2009}$, $\alpha_{(method=i)}^{2010}$ are obtained. Then, the three different short-term alpha estimators are compared. The comparison is based on the alphas' magnitude as well as on the classification and ranking of funds implied by each alpha throughout each of the seven years.

Next, the implied 2004-2007 alpha is calculated:

$$\alpha_{(method=i)}^{2004-2007} = (1+\alpha_{(method=i)}^{2004})^{12} * (1+\alpha_{(method=i)}^{2005})^{12} * (1+\alpha_{(method=i)}^{2006})^{12}$$

$$* (1+\alpha_{(method=i)}^{2007})^{12}) - 1, \tag{9}$$

and the implied 2008-2010 alpha is also calculated:

$$\alpha_{(method=i)}^{2008-2010} = (1+\alpha_{(method=i)}^{2008})^{12} * (1+\alpha_{(method=i)}^{2009})^{12} * (1+\alpha_{(method=i)}^{2010})^{12} - 1. \tag{10}$$

Then, the 2004-2007 and the 2008-2010 calculated alphas are compared with the alphas as extracted from a regression run over 2004-2007 and over 2008-2010, respectively. The comparison is based on the alphas' magnitude as well as on the classification and ranking of funds implied by each alpha, and it examines whether the three short-term alpha estimation methods yield performance scores and rankings that are consistent with their long-term counterparts.

3.2.1 The Regression Models

Three different regression models are applied for evaluating a fund's alpha:

1. The CAPM regression is:

$$R_{P,t} - R_{f,t} = \alpha + \beta(R_{M,t} - R_{f,t}) + \varepsilon_t. \tag{11}$$

The excess market return is based on the value-weight return on all NYSE, AMEX, and NASDAQ stocks (from CRSP) and on the one-month Treasury bill rate (from Ibbotson Associates).

2. The Fama and French three-factor model is:

$$R_{P,t} - R_{f,t} = \alpha + \beta_1(R_{M,t} - R_{f,t}) + \beta_2 HML_t + \beta_3 SMB_t + \varepsilon_t. \tag{12}$$

The factors are: 1) the excess market return is based on the value-weighted return on all NYSE, AMEX, and NASDAQ stocks (from CRSP) and on the one-month Treasury bill rate (from Ibbotson Associates), 2) the performance of value stocks relative to growth stocks (HML, High Minus Low), and 3) the performance of small stocks relative to big stocks (SMB, Small Minus Big), based upon the Fama-French Portfolios.

3. Carhart's (1997) four-factor model is:

$$R_{P,t} - R_{f,t} = \alpha + \beta_1(R_{M,t} - R_{f,t}) + \beta_2 HML_t + \beta_3 SMB_t + \beta_4 MOM_t + \varepsilon_t. \tag{13}$$

The model is based on the Fama and French three-factor model and on an additional momentum factor (MOM) of Carhart (1997) which is the average return of the two highest portfolios for the prior period, minus the average return of the two lowest portfolios for the prior period.

3.3 Results

This subsection examines the following methods for calculating a fund's alpha over a short horizon: 1) the differences method as suggested in this paper, 2) the out-of-sample alpha method, and 3) the one-year regression method. These methods are explained in detail in section 2. The results section examines and reports the degree of similarity between these short-term alpha estimators as well as their congruence with the long-term alpha.

3.3.1 Short-Term Performance Evaluation

For 200X, X=4 . . . 10, each of the three methods above yields a yearly-alpha. Let the 200X alpha extracted from the one-year regression method be $\alpha^{200X}_{(method=1)}$. Let the 200X alpha extracted from the out-of-sample alpha method be $\alpha^{200X}_{(method=2)}$. Let the 200X alpha extracted from the differences method be $\alpha^{200X}_{(method=3)}$.

Alpha is calculated for each year (2004 to 2010) for each of the three short-term alpha estimators and for each of the three regression frameworks - 63 alphas for each fund. The funds are ranked and classified based on each alpha. Then, for each year and for each regression model, the degree of similarity between the three short-term estimates is compared. Table 3 reports the average comparison results of 2004-2010. The average data sample comprises 1112.6 funds.

Table 3: Short-Term Alpha Estimators Comparison

Panel A: Correlations Between Short-Term Alpha Estimators

CAPM

	REGRESSION	DIFFERENCES
OUT-OF-SAMPLE	86.2%	97.8%
REGRESSION		89.4%

Three-Factor Model

	REGRESSION	DIFFERENCES
OUT-OF-SAMPLE	66.8%	93.0%
REGRESSION		79.4%

Four-Factor Model

	REGRESSION	DIFFERENCES
OUT-OF-SAMPLE	54.8%	90.1%
REGRESSION		69.3%

Panel B: Signs Differences Among The Methods

		REGRESSION	DIFFERENCES
OUT-OF-SAMPLE	CAPM	15%	5%
	Three-Factor Model	26%	11%
	Four-Factor Model	33%	13%
REGRESSION	CAPM		12%
	Three-Factor Model		21%
	Four-Factor Model		26%

Panel C: Ranks Differences (More Than A Decile) Among The Methods

		REGRESSION	DIFFERENCES
OUT-OF-SAMPLE	CAPM	36%	10%
	Three-Factor Model	58%	26%
	Four-Factor Model	65%	32%
REGRESSION	CAPM		29%
	Three-Factor Model		48%
	Four-Factor Model		57%

Panel D: Comparing Best 20 Performing Funds Among The Methods

		REGRESSION	DIFFERENCES
OUT-OF-SAMPLE	CAPM	11.3	15.7
	Three-Factor Model	7.9	14.3
	Four-Factor Model	4.4	12.3
REGRESSION	CAPM		12.6
	Three-Factor Model		10.4
	Four-Factor Model		7.1

Panel E: Comparing Worst 20 Performing Funds Among The Methods

		REGRESSION	DIFFERENCES
OUT-OF-SAMPLE	CAPM	11.4	17.3
	Three-Factor Model	6.7	14.7
	Four-Factor Model	5.6	13.6
REGRESSION	CAPM		12.7
	Three-Factor Model		9.4
	Four-Factor Model		8.4

As reported in panel A, the average correlation between the one-year regression estimator and the out-of-sample alpha estimator is 86.2% for the CAPM, 66.8% for the three-factor model and 54.8% for the four-factor model. The average correlation between the one-year regression estimator and the differences estimator is 89.4% for the CAPM, 79.4% for the three-factor model and 69.3% for the four-factor model. The average correlation between the out-of-sample alpha estimator and the differences estimator is 97.8% for the CAPM, 93% for the three-factor model and 90.1% for the four-factor model.

Next, based on the different alphas implied by the different short-term alpha estimation methods, the funds are classified and ranked. Then, funds classification and ranking are compared and the comparison results are reported in Panels B-E of table 3. Recall that for each fund and using each of the regression models, the short-term alpha is estimated via three different methods each year. Based on its alpha, a fund is classified as either a good fund (where alpha is greater than zero) or a bad fund (where alpha is lower than zero). Thus, each year and for each of the three regression models, a fund is classified three times according to its three different short-term alphas. Then, for each pair of short-term alphas (i.e α_i and α_j), the funds' classifications are compared. If a fund is classified as a good (bad) fund according to both short-term alpha estimation method i and short-term alpha estimation method j, then there is no classification difference between methods i and j. On the other hand, if a fund is classified as a good (bad) fund according to short-term alpha estimation method i, but as a bad (good) fund according to short-term alpha estimation method j, then there is a classification difference between methods i and j. The comparison result, per each fund, is a three (regression models) by three (short-term alpha estimation methods) table filled with either a 1 (a classification difference) or a 0 (a classification agreement). Next, the average of all of the funds' classification difference/agreement tables is taken. Panel B reports the 2004-2010 average frequency of classification differences between the short-term alpha estimation methods, for each of the three regression models.

On average, the classification difference between the one-year regression estimator and the out-of-sample alpha estimator is 15% for the CAPM, 26% for the three-factor model and 33% for the four-factor model. The classification difference between the one-year regression estimator and the differences estimator is 12% for the CAPM, 21% for the three-factor model and 26% for the four-factor model. The classification difference between the out-of-sample alpha estimator and the differences estimator is 5% for the CAPM, 11% for the three-factor model and 13% for the four-factor model.

Panel C reports the ranking differences between the short-term alpha estimation methods. Assume N funds are available. Each year, for each regression model and for each short-term alpha estimation method, the best performing fund with the highest alpha is ranked in the 1st place, the second best performing fund with the second highest alpha is ranked in 2nd place, and so on. Since each year a fund has 9 different yearly-alphas (three regression models multiplied by three short-term alpha estimation methods), all funds are ranked 9 times each year. Consider regression model *a*. Assume fund P is ranked in the kth place according to regression model *a* and short-term alpha estimation method i (denoted as $\alpha_{a,i,P}$), and assume that the fund is ranked in the lth place according to the same regression model *a* and to the short-term alpha estimation method j (denoted as $\alpha_{a,j,P}$). If $|k-l| < 0.1N$ (the ranking difference is less than 10% of the N existing funds) then it says that there is no ranking difference between the short-term alpha estimation methods i

and j. On the other hand, if $|k - l| \geq 0.1N$, then there is a ranking difference between methods i and j. The ranking comparison result, per each fund, is a three (regression models) on three (short-term alpha estimation methods) table. If there is a ranking difference, the cell is filled in with a 1 and if there is a ranking agreement, the cell is filled in with a 0. Next, the average of all funds' ranking differences/agreement tables is taken. Panel C reports the 2004-2010 average frequency of ranking differences between the short-term alpha estimation methods, for each of the three regression models.

On average, the ranking difference between the one-year regression estimator and the out-of-sample alpha estimator is 36% for the CAPM, 58% for the three-factor model and 65% for the four-factor model. The ranking difference between the one-year regression estimator and the differences estimator is 29% for the CAPM, 48% for the three-factor model and 57% for the four-factor model. The ranking difference between the out-of-sample alpha estimator and the differences estimator is 10% for the CAPM, 26% for the three-factor model and 32% for the four-factor model.

For the analysis of Panels D and E, the data sample is narrowed and only funds ranked as the 20 best performing funds or as the 20 worst performing funds are kept. Each year, each regression model and short-term alpha estimation method have their own list of funds included in the 20 best and in the 20 worst performing funds. Then, for each regression model, the funds that appear on both the list of short-term alpha estimation method i and the list of short-term alpha estimation method j of best (worst) performing funds are counted, and the 2004-2010 average numbers are reported in Panel D (E).

For the best (worst) performing funds, the average overlap in classification between the one-year regression estimator and the out-of-sample alpha estimator is, with 20 being the highest possible score, 11.3 (11.4) for the CAPM, 7.9 (6.7) for the three-factor model, and 4.4 (5.6) for the four-factor model. For the best (worst) performing funds, the average overlap in classification between the one-year regression estimator and the differences estimator is 12.6 (12.7) out of 20 for the CAPM, 10.4 (9.4) for the three-factor model, and 7.1 (8.4) for the four-factor model. For the best (worst) performing funds, the average overlap in classification between the out-of-sample alpha estimator and the differences estimator is 15.7 (17.3) out of 20 for the CAPM, 14.3 (14.7) for the three-factor model, and 12.3 (13.6) for the four-factor model.

Comparing the three short-term alpha estimators, there are substantial scores and ranking differences between them, though the correlations between the alpha estimates are high. In addition, as the model gets more complicated, the correlation between the short-term estimators decreases and the classification and ranking differences increase.

3.3.2 The Congruence of Short-Term and Long Term Alphas

Next, the short-term alphas' congruence with the long-term alpha is examined.

The yearly alphas, for each method i, are used for calculating the 2004-2007 alpha as:

$$\alpha_{(method=i)}^{2004-2007} = (1 + \alpha_{(method=i)}^{2004})^{12} * (1 + \alpha_{(method=i)}^{2005})^{12} * (1 + \alpha_{(method=i)}^{2006})^{12}$$

$$* (1 + \alpha_{(method=i)}^{2007})^{12} - 1,$$

(14)

and the 2008-2010 alpha as:

$$\alpha_{(method=i)}^{2008-2010} = (1 + \alpha_{(method=i)}^{2008})^{12} * (1 + \alpha_{(method=i)}^{2009})^{12} * (1 + \alpha_{(method=i)}^{2010})^{12} - 1.$$

(15)

Then, a regression is run over 2004-2007 and over 2008-2010. Let the monthly alpha as extracted from those regression be $\alpha_{M,2004-2007}$ and $\alpha_{M,2008-2010}$, respectively. Based on the monthly alpha as extracted from the 2004-2007 regression, the 2004-2007 alpha is calculated as:

$$\alpha_{2004-2007} = (1 + \alpha_{M,2004-2007})^{48} - 1,$$

(16)

and it is compared with each of the three calculated four-year alphas ($\alpha_{(method=i)}^{2004-2007}, i = 1,2,3$). Based on the monthly alpha as extracted from the 2008-2010 regression, the 2008-2010 alpha is calculated as:

$$\alpha_{2008-2010} = (1 + \alpha_{M,2008-2010})^{36} - 1,$$

(17)

and it is compared with each of the three calculated three-year alphas ($\alpha_{(method=i)}^{2008-2010}, i = 1,2,3$). The comparisons are conducted for each of the three regression models. Table 4 reports the 2004-2007 and 2008-2010 average comparison results. The average data sample comprises 692 funds.

Table 4: Short-Term And Long-Term Alpha Estimators Comparison

Panel A: Short-Term And Long-Term Alpha Estimators Correlations

	DIFFERENCES	REGRESSION	OUT-OF-SAMPLE
CAPM	95.9%	90.0%	92.2%
Three-Factor Model	95.3%	70.4%	87.7%
Four-Factor Model	92.9%	73.2%	84.2%

Panel B: Signs Differences - Short-Term Vs. Long-Term Alpha

	DIFFERENCES	REGRESSION	OUT-OF-SAMPLE
CAPM	8.5%	11.8%	11.1%
Three-Factor Model	9.3%	25.7%	15.9%
Four-Factor Model	12.6%	26.8%	18.6%

Panel C: Ranks Differences (More Than A Decile) - Short-Term Vs. Long-Term

	DIFFERENCES	REGRESSION	OUT-OF-SAMPLE
CAPM	22.2%	35.3%	32.8%
Three-Factor Model	23.4%	58.0%	43.0%
Four-Factor Model	31.9%	56.4%	45.8%

Panel D: Comparing Best 20 Performing Funds - Short-Term Vs. Long-Term Alpha

	DIFFERENCES	REGRESSION	OUT-OF-SAMPLE
CAPM	15.0	13.0	13.0
Three-Factor Model	13.0	7.0	11.0
Four-Factor Model	13.5	9.0	10.0

Panel E: Comparing Worst 20 Performing Funds - Short-Term Vs. Long-Term Alpha

	DIFFERENCES	REGRESSION	OUT-OF-SAMPLE
CAPM	14.0	10.5	12.5
Three-Factor Model	15.5	9.5	13.0
Four-Factor Model	16.0	8.0	14.5

Panel A reports the average correlations between the three calculated 2004-2007 alphas and the alpha extracted from the 2004-2007 regression, and between the three calculated 2008-2010 alphas and the alpha extracted from the 2008-2010 regression. For the CAPM (the three-/four-factor model), the average correlation between the differences estimator and the standard (long-term) alpha is 95.9% (95.3%/ 92.9%). The average correlation between the one-year regression estimator and the standard alpha is 90% (70.4%/ 73.2%). The average correlation between the out-of-sample alpha estimator and the standard alpha is 92.2% (87.7%/ 84.2%). Thus, for all regression models, the standard alpha has a higher correlation with the alpha implied by the differences method, relative to the correlations between the standard alpha and the alpha implied by both the one-year regression method as well as by the out-of-sample alpha method.

Panel B reports the average classification differences between the three calculated 2004-2007 alphas and the alpha extracted from the 2004-2007 regression, and between the three calculated 2008-2010 alphas and the alpha extracted from the 2008-2010 regression, for each of the three regression models. The reported classification differences are constructed based on the methodology described in subsection 3.3.1. In general, the differences method generates the smallest differences between the short-term and long-term measures.

Panel C reports the average ranking differences between the three calculated 2004-2007 alphas and the alpha extracted from the 2004-2007 regression, and between the three calculated 2008-2010 alphas and the alpha extracted from the 2008-2010 regression, for each of the three regression models. The reported ranking differences are constructed based on the methodology described in subsection 3.3.1. Again, the differences method minimizes the disparity between long-term and short-term measures.

Panels D and E report the average comparisons of the 20 best and worst performing funds between the three calculated 2004-2007 alphas and the alpha extracted from the 2004-2007 regression, and between the three calculated 2008-2010 alphas and the alpha extracted from the 2008-2010 regression, for each of the three regression models. The reported differences are constructed based on the methodology described in subsection 3.3.1. The differences method again yields the lowest differences.

Comparing the short-term alphas to the standard alpha extracted from a regression run over a three-year or a four-year period, the empirical results show that the differences method yields performance scores and rankings that are the most consistent with their long-term counterparts.

4. ANOTHER APPROACH: SIMULATION

This section creates a simulated database of fund returns with known statistical properties. Then,it estimates the short-term alpha using the out-of-sample alpha method estimator and the differences method estimator, and compares the estimates derived by the two methods. Based on the previous analysis of the methods, it is expected that the differences method would give the best estimate of the short-term alpha.

The framework assumes a DGP such that the fund alpha, the fund beta, and the fund idiosyncratic noise vary periodically. Consider a four-year period. For each simulated fund ($P = 1,...,1000$) and month ($t = 1,...,48$) the fund alpha (α_t), the fund beta (β_t), the fund idiosyncratic noise (ε_t) and the benchmark's monthly return ($R_{M,t}$) are simulated. Then, the fund return is calculated as:

$$R_{P,t} = \alpha_t + \beta_t R_{M,t} + \varepsilon_t.$$

(18)

The benchmark monthly return is simulated as $R_{M,t} \sim N(1\%, 2\%^2)$. This specification is based on the average value and variance of the 1994-2007 following benchmarks returns: S&P500, Russell 1000, Russell 1000 Growth, Russell 1000 Value, Russell Midcap, Russell Midcap Growth, Russell Midcap Value, Russell 2000, Russell 2000 Growth, and Russell 2000 Value.

The fund monthly idiosyncratic noise is simulated as $\varepsilon_t \sim N(-0.17\%, 0.5\%^2)$, the fund monthly alpha as $\alpha_t \sim N(-0.01\%, 0.1\%^2)$ with a serial correlation of 0.034 and the fund monthly beta as $\beta_t \sim N(1.05, 20.51\%^2)$ with a serial correlation of 0.573, following Knuth's methodology for simulating serial correlation (see Knuth (1981)). These specifications are based on the 2001-2007 analysis of 6620 non-specialized open-end equity funds. The funds' monthly betas and alphas are extracted from monthly regressions of the funds' returns on the market return (the CAPM), run over daily data. The monthly epsilons are calculated as the fund monthly return minus the monthly alpha, minus the monthly beta multiplied by the monthly benchmark return ($\varepsilon_t = R_{P,t} - \alpha_t - \beta_t R_{M,t}$). Then, the average value, variance, and serial correlation for alpha, beta, and epsilon are calculated.

The out-of-sample alpha method and the differences method are applied for estimating the fourth year alpha. Then, the average fourth year simulated alpha ($AVG(\alpha_t \mid t = 37..48)$) is calculated and compared with the alpha implied by the out-of-sample alpha method and with the alpha implied by the differences method. This methodology is repeated 1000 times.

Table 5: Simulation Resultsof The Short-Term Alpha Estimation Methods

	OUT-OF-SAMPLE METHOD	ALPHA DIFFERENCES METHOD
AVERAGE($\alpha_{estimator} - \alpha_{from-data}$)	-0.200%	-0.152%
AVERAGE($\mid\alpha_{estimator} - \alpha_{from-data}\mid$)	0.244%	0.203%
% Better Estimator	38%	62%

Table 5 summarizes the simulation results. The results suggest that the differences method estimates the fourth-year simulated alpha better than the out-of-sample alpha method as follows: 1) on average, the difference between the differences method's estimator and the simulated fourth-year alpha is lower than the difference between the out-of-sample alpha method's estimator and the simulated fourth-year alpha, 2) the average absolute value difference between the differences method's estimator and the simulated fourth-year alpha is lower than the average absolute value difference between the out-of-sample alpha method's estimator and the simulated fourth-year alpha, and 3) the number of the simulations in which the differences estimator is the better estimator for the simulated fourth-year alpha is almost twice the number of simulations in which the out-of-sample alpha estimator is the better estimator for the simulated fourth-year alpha, where the better estimator is the one that yields that smallest difference between itself and the simulated fourth-year alpha. Thus, the simulation results support the superiority of the differences method.

5. CONCLUSION

In this paper, a new methodology is suggested for evaluating short-term alpha: the differences method. The intuition behind the differences method is derived from the forward rate calculation. The differences method, as opposed to the widely used out-of-sample alpha method, does not need to assume that the betas are constant over a short period of time.

Three different methods are applied and compared for evaluating short-term excess-returns: the differences method, the out-of-sample alpha method and the one-year regression method. Substantial score and ranking differences are found between the short-term estimators, although the correlations between the alpha estimates are high. In addition, as the model gets more complicated, the correlation between the short-term alpha estimators decreases and the classification and ranking differences increase.

The short-term alpha estimation methods are analyzed and compared based on aspects of expected value and variance. The analysis implies that all three methods' estimators have the same expected value, and that the differences method's estimator has the lowest variance. Thus, the conclusion is that the differences method is the better short-term alpha evaluation method of the three methods examined in this paper. In addition, a simulation supports this result. The simulation suggests that the differences method better estimates the short-term alpha since its estimator yields, on average, the smallest difference (whether the absolute value or not) between itself and the simulated alpha. Finally, the new methodology also yields performance scores and rankings that are the most consistent with their long-term counterparts. The conclusion is that the differences method, as suggested in this paper, should be adopted.

REFERENCES

Bauer, R., Cosemans, M. and Eichholtz, P. (2007), The performance and persistence of individual investors: rational agents or tulip maniacs? Working Paper (Maastricht University).

Ben-Rephael, A., Kadan, O. and Wohl, A. (2010), The diminishing liquidity premium, Working Paper.

Berk, J. B. and Green, R. C. (2004), Mutual fund flows and performance in rational markets, Journal of Political Economy, Vol. 112, pp. 1269-1295.

Bollen, N. and Busse, J. A. (2005), Short-term persistence in mutual fund performance, Review of Financial Studies, Vol. 18, pp. 569-597.

Brennan, M. J., Chordia, T. and Subrahmanyam, A. (1998), Alternative factor specifications, security characteristics and the cross-section of expected stock returns, Journal of Financial Economics, Vol. 49, pp. 345-373.

Brennan, M. J., Chordia, T., Subrahmanyam, A. and Tong, Q. (2009), Sell-side illiquidity and the cross-section of expected stock returns, Working Paper.

Brown, K. C., Harlow, W. V. and Starks, L. T. (1996), Of tournaments and temptations: an analysis of managerial incentives in the mutual fund industry, Journal of Finance, Vol. 51, pp. 85-110.

Brown, S. J. and Goetzmann, W. N. (1995), Performance persistence, Journal of Finance, Vol. 50, pp. 679-698.

Brown, S. J., Goetzmann, W. N., Ibbotson, R. G. and Ross, S. A. (1992), Survivorship bias in performance studies, Review of Financial Studies, Vol. 5, pp. 553-580.

Busse, J. A. (2001), Another look at mutual fund tournaments, Journal of Financial and Quantitative Analysis, Vol. 36, pp. 53-73.

Carhart, M. M. (1997), On persistence in mutual funds performance, Journal of Finance, Vol. 52, pp. 57-82.

Carlson, R. S. (1970), Aggregate performance of mutual funds, 1948-1967, Journal of Financial and Quantitative Analysis, Vol. 5, pp. 1-32.

Chance, D. M. (2011), The alpha bias in asset allocation performance measurement. Working Paper (Available at SSRN: http://ssrn.com/abstract=1866301).

Chen, H. L., Jegadeesh, N. and Wermers, R. (2000), The value of active mutual fund management: an examination of the stockholdings and trades of fund managers, Journal of Financial and Quantitative Analysis, Vol. 35, pp. 343-368.

Chevalier, J. and Ellison, G. (1997), Risk-taking by mutual funds as a response to incentives, Journal of Political Economy, Vol. 105, pp. 1167-1200.

Chordia, T., Huh, S. W. and Subrahmanyam, A. (2009), Theory-based illiquidity and asset pricing, Review of Financial Studies, Vol. 22, pp. 3629-3668.

Chordia, T. and Shivakumar, L. (2006), Earnings and price momentum, Journal of Financial Economics, Vol. 80, pp. 627-656.

Chordia, T., Subrahmanyam, A. and Anshuman, V. R. (2001), Trading activity and expected stock returns, Journal of Financial Economics, Vol. 59, pp. 3-32.

Connor, G. and Korajczyk, R. A. (1988), Risk and return in an equilibrium APT: application of a new test methodology, Journal of Financial Economics, Vol. 21, pp. 255-289.

Cremers, M. and Petajisto, A. (2009), How active is your fund manager? A new measure that predicts performance, Review of Financial Studies, Vol. 22, pp. 3329-3365.

Elton, E. J., Gruber, M. J. and Blake, C. R. (1996), Survivorship bias and mutual fund performance, Review of Financial Studies, Vol. 9, pp. 1097-1120.

Fama, E. F. and French, K. R. (1993), Common risk factors in the returns on stocks and bonds, Journal of Financial Economics, Vol. 33, pp. 3-56.

Fama, E. F. and French, K. R. (1996), Multifactor explanations of asset pricing anomalies, Journal of Finance, Vol. 51, pp. 55-84.

Ferson, W. E. and Harvey, C. R. (1999), Conditioning variables and the cross section of stock returns, Journal of Finance, Vol. 54, pp. 1325-1360.

Ferson, W. E. and Schadt, R. W. (1996), Measuring fund strategy and performance in changing economic conditions, Journal of Finance, Vol. 51, pp. 425-461.

Goetzmann, W. N. and Ibbotson, R. G. (1994), Do winners repeat? Journal of Portfolio Management, Vol. 20, pp. 9-18.

Goriaev, A. P., Nijman, T. E. and Werker, B. J. (2005), Yet another look at mutual fund tournaments, Journal of Empirical Finance, Vol. 12, pp. 127-137.

Graham, J. R. and Campbell, H. R. (1997), Grading the performance of market-timing newsletters, Financial Analysts Journal, Vol. 53, pp. 54-66.

Grinblatt, M. and Titman, S. (1992), The persistence of mutual fund performance, Journal of Finance, Vol. 47, pp. 1977-1984.

Grinblatt, M., Titman, S. and Wermers, R. (1995), Momentum investment strategies, portfolio performance and herding: a study of mutual fund behavior, American Economic Review, Vol. 85, pp. 1088-1105.

Gruber, M. J. (1996), Another puzzle: the growth in actively managed mutual funds, Journal of Finance, Vol. 51, pp. 783-810.

Hendricks, D., Patel, J. and Zeckhauser, R. (1993), Hot hands in mutual funds: short run persistence of relative performance, 1974-1988, Journal of Finance, Vol. 48, pp. 93-130.

Hendricks, D., Patel, J. and Zeckhauser, R. (1997), The j-shape of performance persistence given survivorship bias, Review of Economics and Statistics, Vol. 79, pp. 161-166.

Hunter, D., Kandel, E., Kandel, S. and Wermers, R. (2009), Endogenous benchmarks, Working Paper (University of Maryland).

Ippolito, R. A. (1989), Efficiency with costly information: a study of mutual fund performance, 1965-1984, Quarterly Journal of Economics, Vol. 104, pp. 1-23.

Ippolito, R. A. (1992), Consumer reaction to measures of poor quality: evidence from the mutual fund industry, Journal of Law and Economics, Vol. 35, pp. 45-70.

Jegadeesh, N. and Titman, S. (1993), Returns to buying winners and selling losers: implications for stock market efficiency, Journal of Finance, Vol. 48, pp. 65-91.

Jensen, M. C. (1968), The performance of mutual funds in the period 1945-1964, Journal of Finance, Vol. 23, pp. 389-416.

Kaplan, S. N. and Schoar, A. (2005), Private equity performance: returns, persistence and capital flows, Journal of Finance, Vol. 60, pp. 1791-1823.

Knuth, D. E. (1981), The Art of Computer Programming: Semi-Numerical Algorithms, volume 2, Addison-Wesley Publishing Co.

Kosowski, R., Timmermann, A., Wermers, R. and White, H. (2006), Can mutual fund stars really pick stocks? New evidence from a bootstrap analysis, Journal of Finance, Vol. 61, pp. 2551-2595.

Kumar, A. (2009), Dynamic style preferences of individual investors and stock returns, Journal of Financial and Quantitative Analysis, Vol. 44, pp. 607-640.

Malkiel, B. G. (1995), Returns from investing in equity mutual funds 1971 to 1991, Journal of Finance, Vol. 50, pp. 549-572.

Otten, R. and Bams, D. (2002), European mutual fund performance, European Financial Management, Vol. 8, pp. 75-101.

Ross, S. A. (1976), The arbitrage theory of capital asset pricing, Journal of Economic Theory, Vol. 13, pp. 341-360.

Sharpe, W. F. (1964), Capital asset prices: a theory of market equilibrium under conditions of risk, Journal of Finance, Vol. 19, pp. 425-442.

Sharpe, W. F. (1966), Mutual fund performance, Journal of Business 39, pp. 119-138.

Shukla, R. and Trzcinka, C. (1994), Persistent performance in the mutual fund market: tests with funds and investment advisors, Review of Quantitative Finance and Accounting, Vol. 4, pp. 115-135.

Sirri, E. R. and Tufano, P. (1998), Costly search and mutual fund flows, Journal of Finance, Vol. 53, pp. 1589-1622.

Spiegel, M. and Wang, X. (2006), Cross-sectional variation in stock returns: liquidity and idiosyncratic risk, Working Paper (Yale University).

Treynor, J. L. (1965), How to rate management of investment funds, Harvard Business Review, Vol. 43, pp. 63-75.

Wermers, R. (2004), Is money really smart? New evidence on the relation between mutual fund flows, manager behavior and performance persistence, Working Paper (University of Maryland).

Zheng, L. (1999), Is money smart? A study of mutual fund investors' fund selection ability, Journal of Finance, Vol. 54, pp. 901-933.

Appendix

Proof of Results

Proof of result 1:. The expected value of beta's estimator is

$$E(\hat{\beta}) = E(\frac{\sum(x_t - \bar{x})(y_t - \bar{y})}{\sum(x_t - \bar{x})^2}) =$$

(19)

$$= E(\frac{\sum x_t y_t - n\overline{xy}}{\sum x_t^2 - n\bar{x}^2}) =$$

$$= E(\frac{\sum x_t(\alpha_t + \beta_t x_t + u_t) - n\bar{x}(\sum \frac{\alpha_t}{n} + \sum \frac{\beta_t x_t}{n} + \sum \frac{u_t}{n})}{\sum x_t^2 - n\bar{x}^2}) =$$

$$= E(\frac{\sum(x_t\alpha_t + x_t\beta_t x_t + x_t u_t) - \bar{x}\sum\alpha_t - \bar{x}\sum\beta_t x_t - \bar{x}\sum u_t}{\sum x_t^2 - n\bar{x}^2}).$$

Recall that x_t is constant at time t and that $E(u_t) = 0$. Thus,

$$E(\hat{\beta}) = \frac{1}{(\sum x_t^2 - n\bar{x}^2)}[\sum x_t E(\alpha_t) + \sum x_t^2 E(\beta_t) - \bar{x}\sum E(\alpha_t) - \bar{x}\sum E(\beta_t)x_t] =$$

(20)

$$= \frac{\alpha(\sum x_t - n\bar{x})}{(\sum x_t^2 - n\bar{x}^2)} + \frac{\beta(\sum x_t^2 - \bar{x}\sum x_t)}{(\sum x_t^2 - n\bar{x}^2)}.$$

Note that $\sum x_t - n\bar{x} = 0$ and $\sum x_t^2 - \bar{x}\sum x_t = \sum x_t^2 - n\bar{x}^2$. Thus, $E(\hat{\beta}) = \beta$.

Proof of result 2:. The expected value of alpha's estimator is

$$E(\hat{\alpha}) = E(\bar{y} - \hat{\beta}\bar{x}) =$$

(21)

$$= E(\sum\frac{\alpha_t}{n} + \sum\frac{\beta_t x_t}{n} + \sum\frac{u_t}{n} - \frac{\sum(x_t - \bar{x})(y_t - \bar{y})}{\sum(x_t - \bar{x})^2}\bar{x}) =$$

$$= E(\sum\frac{\alpha_t}{n} + \sum\frac{\beta_t x_t}{n} + \sum\frac{u_t}{n} - \frac{\sum x_t y_t - n\overline{xy}}{\sum(x_t - \bar{x})^2}\bar{x}) =$$

$$= E(\sum \frac{\alpha_t}{n} + \sum \frac{\beta_t x_t}{n} + \sum \frac{u_t}{n} - \frac{\sum x_t(\alpha_t + \beta_t x_t + u_t) - n\bar{x}(\sum \frac{\alpha_t}{n} + \sum \frac{\beta_t x_t}{n} + \sum \frac{u_t}{n})}{\sum (x_t - \bar{x})^2} \bar{x}).$$

Recall that x_t is constant at time t and that $E(u_t) = 0$. Thus

$$E(\hat{\alpha}) = \sum \frac{E(\alpha_t)}{n} + \sum \frac{E(\beta_t)x_t}{n} - \frac{\bar{x}}{\sum (x_t - \bar{x})^2} [\sum x_t E(\alpha_t) + \sum x_t^2 E(\beta_t)$$

(22)

$$-n\bar{x} \sum \frac{E(\alpha_t)}{n} - n\bar{x} \sum \frac{E(\beta_t)x_t}{n}] =$$

$$= \alpha + \beta\bar{x} - \frac{\bar{x}}{\sum (x_t - \bar{x})^2} [\alpha \sum x_t + \beta \sum x_t^2 - n\bar{x}\alpha - n\bar{x}\beta \sum \frac{x_t}{n}].$$

Note that $\alpha \sum x_t = n\bar{x}\alpha$, so

$$E(\hat{\alpha}) = \alpha + \beta\bar{x} - \frac{\bar{x}}{\sum (x_t - \bar{x})^2} [\beta(\sum x_t^2 - \bar{x} \sum x_t)].$$

(23)

In addition, note that $(\sum x_t^2 - \bar{x} \sum x_t) = \sum (x_t - \bar{x})^2$, so

$$E(\hat{\alpha}) = \alpha + \beta\bar{x} - \bar{x}\beta$$

(24)

$$= \alpha.$$

Proof of result 3:. Consider the out-of-sample alpha method. Let the alpha evaluated by the out-of-sample alpha method be $\alpha_{(2)}^4$. Then $\alpha_{(2)}^4$ is evaluated as follows:

$$\alpha_{(2)}^4 = \bar{y}^4 - \hat{\beta}^{1-3}\bar{x}^4,$$

(25)

where $\bar{y}^4$ is the fourth-year average security return, $\hat{\beta}^{1-3}$ is the beta extracted from a regression of the security excess return over the benchmark excess return run over years 1 to 3, and $\bar{x}^4$ is the fourth-year average benchmark return. Adding and subtracting $\hat{\beta}^4\bar{x}^4$ to and from that equation gives

$$\alpha_{(2)}^4 = \bar{y}^4 - \hat{\beta}^{1-3}\bar{x}^4 =$$

(26)

$$= \bar{y}^4 - \hat{\beta}^{1-3}\bar{x}^4 - \hat{\beta}^4\bar{x}^4 + \hat{\beta}^4\bar{x}^4.$$

Note that $\bar{y}^4 - \hat{\beta}^4\bar{x}^4$ is, by definition, $\hat{\alpha}^4$. Therefore

$$\alpha^4_{(2)} = \bar{y}^4 - \hat{\beta}^4\bar{x}^4 + \bar{x}^4(\hat{\beta}^4 - \hat{\beta}^{1-3}) =$$

$$= \hat{\alpha}^4 + \bar{x}^4(\hat{\beta}^4 - \hat{\beta}^{1-3}).$$

(27)

The expected value of $\alpha^4_{(2)}$ is:

$$E(\alpha^4_{(2)}) = E(\hat{\alpha}^4 + \bar{x}^4(\hat{\beta}^4 - \hat{\beta}^{1-3})) =$$

(28)

$$= E(\hat{\alpha}^4) + E(\bar{x}^4(\hat{\beta}^4 - \hat{\beta}^{1-3})) =$$

$$= E(\hat{\alpha}^4) + \bar{x}^4 E(\hat{\beta}^4 - \hat{\beta}^{1-3}) =$$

$$= E(\hat{\alpha}^4) + \bar{x}^4(E(\hat{\beta}^4) - E(\hat{\beta}^{1-3})) =$$

$$= E(\hat{\alpha}^4) + \bar{x}^4 E(\beta - \beta) =$$

$$= E(\hat{\alpha}^4) =$$

$$= \alpha.$$

The variance of $\alpha^4_{(2)}$ is:

$$VAR(\alpha^4_{(2)}) = E[(\alpha^4_{(2)} - E(\alpha^4_{(2)}))^2] =$$

(29)

$$= E((\hat{\alpha}^4 + \bar{x}^4(\hat{\beta}^4 - \hat{\beta}^{1-3}) - \alpha)^2) =$$

$$= E((\hat{\alpha}^4)^2 + (\bar{x}^4)^2(\hat{\beta}^4 - \hat{\beta}^{1-3})^2 + \alpha^2$$

$$+ 2\hat{\alpha}^4\bar{x}^4(\hat{\beta}^4 - \hat{\beta}^{1-3}) - 2\alpha\hat{\alpha}^4 - 2\alpha\bar{x}^4(\hat{\beta}^4 - \hat{\beta}^{1-3})) =$$

$$= E((\hat{\alpha}^4)^2) + E((\bar{x}^4)^2(\hat{\beta}^4 - \hat{\beta}^{1-3})^2) + E(\alpha^2)$$

$$+ E(2\hat{\alpha}^4\bar{x}^4(\hat{\beta}^4 - \hat{\beta}^{1-3})) - E(2\alpha\hat{\alpha}^4) - E(2\alpha\bar{x}^4(\hat{\beta}^4 - \hat{\beta}^{1-3})).$$

Recall that x_t is known at time t. Thus

$$VAR(\alpha^4_{(2)}) = E((\hat{\alpha}^4)^2) + (\bar{x}^4)^2 E((\hat{\beta}^4 - \hat{\beta}^{1-3})^2) + \alpha^2$$

(30)

$$+2\bar{x}^4 E(\hat{\alpha}^4)E(\hat{\beta}^4 - \hat{\beta}^{1-3}) - 2\alpha E(\hat{\alpha}^4) - 2\alpha\bar{x}^4 E(\hat{\beta}^4 - \hat{\beta}^{1-3}).$$

By definition, $E(x^2) = VAR(x) + E(x)^2$. In addition, recall that $E(\hat{\beta}^4 - \hat{\beta}^{1-3}) = 0$. Thus

$$VAR(\alpha_{(2)}^4) = VAR(\hat{\alpha}^4) + E(\hat{\alpha}^4)^2 + (\bar{x}^4)^2(VAR(\hat{\beta}^4 - \hat{\beta}^{1-3}) + E(\hat{\beta}^4 - \hat{\beta}^{1-3})^2) + \alpha^2 - 2\alpha^2 =$$
(31)

$$= VAR(\hat{\alpha}^4) + \alpha^2 + (\bar{x}^4)^2(VAR(\hat{\beta}^4 - \hat{\beta}^{1-3})) + \alpha^2 - 2\alpha^2 =$$

$$= VAR(\hat{\alpha}^4) + (\bar{x}^4)^2(VAR(\hat{\beta}^4 - \hat{\beta}^{1-3})).$$

Proof of result 4:. Consider the differences method. Let the alpha evaluated by the differences method be $\alpha_{(3)}^4$. Then $\alpha_{(3)}^4$ is evaluated as:

$$\alpha_{(3)}^4 = \frac{(1 + \hat{\alpha}^{1-4})^4}{(1 + \hat{\alpha}^{1-3})^3} - 1,$$
(32)

where $\hat{\alpha}^{1-3}$ is the alpha extracted from a regression of the security excess returns over the benchmark excess returns run over years 1 to 3 and $\hat{\alpha}^{1-4}$ is the alpha extracted from a regression of the security excess returns over the benchmark excess returns run over years 1 to 4. The Taylor series expansion is used for calculating the expected value and variance of $\alpha_{(3)}^4$. The Taylor series expansion of $f(x,y)$ about the values (x_0, y_0) is given by:

$$f(x,y) \approx f(x_0, y_0) + \frac{\partial f(x,y)}{\partial x}\Big|_{(x_0, y_0)}(x - x_0) + \frac{\partial f(x,y)}{\partial y}\Big|_{(x_0, y_0)}(y - y_0) + \dots.$$

Letting $x_0 = E(x) = \bar{x}$, the mean of x, and letting $y_0 = E(y) = \bar{y}$, the mean of y, a Taylor expansion series of $f(x,y)$ about $(\bar{x}, \bar{y})$ gives the approximation

$$f(x,y) \approx f(\bar{x}, \bar{y}) + \frac{\partial f(\bar{x}, \bar{y})}{\partial x}(x - \bar{x}) + \frac{\partial f(\bar{x}, \bar{y})}{\partial y}(y - \bar{y}).$$

The expected value of $f(x,y)$, approximated by the Taylor series expansion, is

$$E(f(x,y)) \approx E(f(\bar{x}, \bar{y}) + \frac{\partial f(\bar{x}, \bar{y})}{\partial x}(x - \bar{x}) + \frac{\partial f(\bar{x}, \bar{y})}{\partial y}(y - \bar{y})) =$$

$$= f(\bar{x}, \bar{y}) + \frac{\partial f(\bar{x}, \bar{y})}{\partial x}E(x - \bar{x}) + \frac{\partial f(\bar{x}, \bar{y})}{\partial y}E(y - \bar{y}).$$

Since $E(x - \bar{x}) = 0$ and $E(y - \bar{y}) = 0$, then

$$E(f(x,y)) \approx f(\bar{x}, \bar{y}).$$

The variance of $f(x,y)$, approximated by the Taylor series expansion, is

$$VAR(f(x,y)) \approx VAR[f(\bar{x}, \bar{y}) + \frac{\partial f(\bar{x}, \bar{y})}{\partial x}(x - \bar{x}) + \frac{\partial f(\bar{x}, \bar{y})}{\partial y}(y - \bar{y})] =$$

$$= (\frac{\partial f(\bar{x}, \bar{y})}{\partial x})^2 VAR(x) + (\frac{\partial f(\bar{x}, \bar{y})}{\partial y})^2 VAR(y) + 2\frac{\partial f(\bar{x}, \bar{y})}{\partial x}\frac{\partial f(\bar{x}, \bar{y})}{\partial y}COV(x,y).$$

Next, the expected value and the variance of the two variable function approximated by the Taylor series expansion are applied to evaluate the expected value and the variance of $\alpha_{(3)}^4$. Recall that:

$$\alpha_{(3)}^4 = \frac{(1 + \hat{\alpha}^{1-4})^4}{(1 + \hat{\alpha}^{1-3})^3} - 1,$$

$$(33)$$

and that:

$$E(\hat{\alpha}^{1-4}) = E(\hat{\alpha}^{1-3}) = \alpha.$$

$$(34)$$

The expected value of $\alpha_{(3)}^4$ is:

$$E(\alpha_{(3)}^4) \approx \frac{(1 + \hat{\alpha}^{1-4})^4}{(1 + \hat{\alpha}^{1-3})^3}\Big|_{(\alpha,\alpha)} - 1 =$$

$$(35)$$

$$= \frac{(1 + \alpha)^4}{(1 + \alpha)^3} - 1 =$$

$$= \alpha.$$

The variance of $\alpha_{(3)}^4$ is:

$$VAR(\alpha_{(3)}^4) \approx (\frac{\partial \alpha_{(3)}^4(\hat{\alpha}^{1-4}, \hat{\alpha}^{1-3})}{\partial \hat{\alpha}^{1-4}})^2\Big|_{(\alpha,\alpha)} VAR(\hat{\alpha}^{1-4}) + (\frac{\partial \alpha_{(3)}^4(\hat{\alpha}^{1-4}, \hat{\alpha}^{1-3})}{\partial \hat{\alpha}^{1-3}})^2\Big|_{(\alpha,\alpha)} VAR(\hat{\alpha}^{1-3})$$

$$(36)$$

$$+ 2\frac{\partial \alpha_{(3)}^4(\hat{\alpha}^{1-4}, \hat{\alpha}^{1-3})}{\partial \hat{\alpha}^{1-4}}\Big|_{(\alpha,\alpha)} \frac{\partial \alpha_{(3)}^4(\hat{\alpha}^{1-4}, \hat{\alpha}^{1-3})}{\partial \hat{\alpha}^{1-3}}\Big|_{(\alpha,\alpha)} COV(\hat{\alpha}^{1-4}, \hat{\alpha}^{1-3}) =$$

$$= (4)^2 (\frac{(1+\alpha)^3}{(1+\alpha)^3})^2 VAR(\hat{\alpha}^{1-4}) + (-3)^2 (\frac{(1+\alpha)^4}{(1+\alpha)^4})^2 VAR(\hat{\alpha}^{1-3})$$

$$+ 2*4\frac{(1+\alpha)^3}{(1+\alpha)^3} * (-3)\frac{(1+\alpha)^4}{(1+\alpha)^4} COV(\hat{\alpha}^{1-4}, \hat{\alpha}^{1-3}) =$$

$$= 16 VAR(\hat{\alpha}^{1-4}) + 9 VAR(\hat{\alpha}^{1-3}) - 24 COV(\hat{\alpha}^{1-4}, \hat{\alpha}^{1-3}).$$

$\hat{\alpha}^{1-3}$ and $\hat{\alpha}^{1-4}$ are estimated over coinciding periods. In addition, since the alphas are estimated over long enough periods (a three-year period (years 1 to 3) and a coinciding four-year period (years 1 to 4)) it is safe to assume that $VAR(\hat{\alpha}^{1-4}) \approx VAR(\hat{\alpha}^{1-3})$ and that $VAR(\hat{\alpha}^{1-4}) \approx VAR(\hat{\alpha}^{1-3}) \approx COV(\hat{\alpha}^{1-4}, \hat{\alpha}^{1-3})$.

Thus,

$$VAR(\alpha_{(3)}^4) \approx 16 VAR(\hat{\alpha}^{1-4}) + 9 VAR(\hat{\alpha}^{1-3}) - 24 COV(\hat{\alpha}^{1-4}, \hat{\alpha}^{1-3}) \approx VAR(\hat{\alpha}^{1-4})$$

or

$$\approx VAR(\hat{\alpha}^{1-3}).$$