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THE SUSPICION OF MANIPULATION IN BITCOIN RETURNS: AN INVESTIGATION WITH BENFORD'S LAW

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Omer Iskenderoglu¹, Nazif Ayyildiz²

¹Omer Halisdemir University, Faculty of Economics and Admisintrative Sciences, Department of Business, Nigde, Turkey.

oiskenderoglu@ohu.edu.tr , ORCID: 0000-0002-3407-1259

²Harran University, Suruc Vocational School, Sanliurfa, Turkey.

nazifayyildiz@harran.edu.tr , ORCID: 0000-0002-7364-8436

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ABSTRACT

Purpose- Bitcoin is a blockchain-based digital currency that can be generated via data mining. Several complex computational methods along with random processes have been utilized in the production of that currency. Nonetheless, it is an important research question whether this process, which have been taking place in a entirely digital platform, involves manipulation. Accordingly, the amount of Bitcoin in circulation would involve manipulation as much as the Bitcoin price and returns do.

Methodology- Related tests are performed to detect compliance with Benford's Law in the analyses conducted on the issue. Distribution frequency of the digits can be determined by Benford's Law in order to determine whether the random digital database is manipulated. In order to detect any possible manipulation in Bitcoin returns, the Chi-Square test is performed

Findings- With the daily Bitcoin price data obtained over the period between 02.02.2012 - 10.02.2020 its found out that Bitcoin prices comply with Benford's Law reference distribution.

Conclusion- According to the results of the analysis, it is concluded that the Bitcoin returns comply with Benford's Law. Therefore, there is no possible manipulation on the Bitcoin returns throughout the study period.

Keywords: Benford's Law, Bitcoin, Detection of Fraud, Digital Analysis, Manipulation. JEL Codes: D41, E42, E31, G12

1. INTRODUCTION

Since the Lydians used gold in trade for more than 2500 years, money had been very crucial as a store of value and a means of exchange for economic and social development. The forms of money have constantly been changing based on technological improvements. The precious metals with specific values associated with their own material contents have been replaced by symbolic base metals and completed with self-valued paper-based notes.

As the revolution of information technology advanced, and especially following the introduction of credit cards in the 1960s, financial transactions became much more virtual (Dodgson, Gann and George 2015, p. 325). Digital money, which has emerged as a new technology in recent years, stands for a medium of exchange in the electronic environment and the means of payment stored in a fully digital format as a measure of value.

The foundation of digital currencies is based on cryptographic improvements. The capacity to secure communication has forced many researchers to create digital currencies, but they did not succeed in their first trials due to their centralization, precious metal support, counterfeiting, and double spending problems. However, these problems have been solved by creating digital signatures in a suitable technological architecture as the ultimate success of decades of developments and technological cryptography practices (Poyser 2017, p. 6).

Digital values that allow cryptographic/cryptographically safe operations and additional virtual money supply are called crypto-currencies. Crypto-currencies are alternative currencies, they are digital, and they are also virtual money. With crypto-currencies, individuals or institutions can expenditures or exchanges as they do with real money. The amount of crypto-

currencies as well as how and when they would be introduced to the market are determined at the very beginning of the system's implementation. No third party or intermediary exists, except for the sender and the recipient, since the system is safe in storing and transferring the cryptocurrencies. Furthermore, no government, institution or company can neither produce cryptocurrencies nor can confiscate anyone's cryptocurrencies (Çarkacıoğlu 2016, p. 8-9).

Upon examining the cryptocurrencies, it is basically possible to consider a distinction between Bitcoin, as the most popular currency, and the others. As a matter of fact, other crypto-currencies that entered the market following Bitcoin, which was the first cryptocurrency, were described as alternative cryptocurrencies since they were inspired by Bitcoin and named as "altcoins". All altcoins serve as alternatives to the Bitcoin. There are two kinds of altcoins. Altcoins, created by utilizing Bitcoin's original open-source protocol, were designed as new coins possessing different features along with alterations in Bitcoin's basic code. Litecoin sets an example of such altcoins. Other types of altcoins include the ones which were not based on the open-source protocol of Bitcoin but have their own protocols and distributed ledgers.

Well-known examples of such altcoins are Ethereum and Ripple (Houben and Snyers 2018, p. 29). As of March 2019, 2,526 altcoins are in circulation, and this amount is subject to increase day by day.

Bitcoin was first coined by Satoshi Nakamoto's article entitled "Bitcoin: A Peer-to-Peer Electronic Cash System" published in 2008. The study, in which Bitcoin was described, explained how the blockchain technology, as the infrastructure of cryptocurrencies, was prepared by the developers and how the system worked. Moreover, the most important feature of the cryptocurrency in the article is that it is based on mathematical rules and based on computer algorithms, rather than trust in the authorities that have the power of printing money (Nakamoto 2008, p. 1-8).

The first Bitcoin transfer was realized between Satoshi Nakamoto and a programmer named Hal Finney on January 12, 2009, after Nakamoto published Bitcoin Whitepaper on January 30, 2008. As of October 2009, the New Liberty Standard has announced the first Bitcoin Exchange rate as \$ 1 = 1,309.03 BTC. As of May 2010, the first shopping with cryptocurrency was carried out in the BitcoinTalk forum along with the payment of 10,000 Bitcoin by Laszlo Hanyecz. In July 2010, the first Bitcoin Stock Exchange, namely, Mt. Gox was established. The market value of the total Bitcoin in circulation as of August 2010 exceeded \$ 1 Million. By February 2011, Bitcoin was equalized to \$ 1 for the first time (\$ 1 = 1 BTC). In April 2011, along with the sale of Bitcoin in Euro and British Pound, the market value reached \$ 10 million. As of March 2013, the market value of Bitcoin exceeded \$ 1 billion. Bitcoin, which exceeded \$ 10,000 as of November 2017, reached \$ 20,000 in December 2017. In March 2021, along with the production of 18,660,000 Bitcoins, 88.8% of totally 21,000,000 Bitcoins were produced. Bitcoin has reached the age of 12 as of January 2021 and has a market value of nearly \$ 1 trillion.

There is a great deal of uncertainty about the extent to which Bitcoin is being used as an exchange instrument and investment asset (Jermann 2018, p. 1). It is suggested that the infrastructure of crypto coins, especially Bitcoin, cannot be manipulated and distorted since a mathematical and technological system is utilized. Nonetheless, the Bitcoin ecosystem, which is claimed to be unmanipulated and undistorted by innovations in technical design, has frequently been a target for financially motivated criminals (Gandal et al. 2017, p. 1). On the other hand, there are three different ways to obtain Bitcoin. The first one is to survey new mines and the second one involves purchasing in a stock exchange and the last one is to accept Bitcoin in Exchange of goods and services (Doğan, Buyrukoğlu and Kutbay 2018, p. 26).

Since the second and third ways are related to the amount of Bitcoin in circulation, there is an increasing number of question marks in minds regarding whether or not newly generated Bitcoins involve any manipulation in terms of quantity and price.

Different digital analyses can be performed by using various techniques on financial market transactions. Digital analysis techniques provide opportunities to simplify and accelerate the course of the analyses. Benford Analysis, one of the most widely used mathematical techniques in economics and finance, looms large with its practicality and low cost. Benford Analysis is based on the fact that the probability of occurrence of digits in randomly formed numbers differs at each place (Benford 1938, p. 551). With Benford's Law, the frequency distribution of digits and digits formed by intervening in the database can be determined. Since the frequency of occurrence of numbers does not comply with Benford's Law; it is revealed that there is a change over the digits data, the data deviate from the natural distribution, and manipulation exists. With the Benford analysis, it can be determined whether or not manipulation in securities traded in capital markets exists.

There are a variety of manipulation methods commonly used in stock exchanges. Although the procedures in these methods usually have a legal appearance, they lead to manipulation whenever they are carried out in a coordinated manner serving a specific purpose. It is also possible to use different methods in each case. Therefore, it is not possible to consider the limited criteria of manipulations (SPK 2003, p. 12). Since the use of the transaction volume in detecting the process-based manipulation may not be sufficient to explain manipulation, therefore it is thought that it would be a more accurate approach to investigate the manipulation of the prices arising in the market as a result of the transactions.

This study aims to determine whether or not any manipulation in the Bitcoin returns exists. To this end, it is examined in detail with the Chi-Square conformity test to determine whether or not the distributions of the daily Bitcoin returns over the period from 02.02.2012 to 10.02.2020 comply with Benford's Law.

Within the scope of the study, and the Bitcoin currency, as well as other cryptocurrencies, are primarily examined. Then, empirical studies in the literature examining manipulation on financial assets and manipulation on a limited number of cryptocurrencies are reviewed. Afterward, the dataset and methodology used are explained, and the findings obtained from the analysis are presented. In the last section, the findings of the analysis are interpreted, and various evaluations are made.

2. LITERATURE REVIEW

Initially, the empirical studies examining the manipulation issue in the securities markets are reviewed. In the last part, practical studies explicating the issue of manipulation in Bitcoin and other cryptocurrencies are introduced.

Ley (1996), one of the seminal studies which was conducted on the manipulation of securities markets, investigated the daily actual returns over the period between January 1900 - June 1993 for the Dow Jones Industrial Average Index (DJIA), and over the period between January 1926 - June 1993 for the Standard and Poor's Index (S&P). According to the results of the study, it is determined that the figures in one-day yield series of stock index indexes are in compliance with Benford's Law. Shengmin and Wenchao (2010), in which the issue of manipulation in index returns was analyzed utilizing different country datasets, examined the distribution of the first digits of two main Chinese stock market indexes (Shanghai Stock Exchange Composite Index and Shenzhen Stock Exchange Component Index) regarding their compliance with Benford's Law. It is concluded that the return series of both indexes practically comply with the Benford law. In addition, the probability of occurrence of the digit "9" in the first digits distribution of the SSE index was found to be higher than expected. Alagöz and Ay (2001), one of the similar studies conducted on the issue in Turkey, tested the data of totally 227 companies trading in Istanbul the Stock Exchange as of 2002 consisting of monthly purchase-sale amounts in US Dollars which vary between the digits "3" and "8" in terms of their compliance with Benford's Law. According to the results of the study, no significant differences were found upon comparing the expected values of Benford's Law with the observed values.

Akkaş (2015a), which was conducted on the issue of manipulation in the daily gold returns, examined the daily gold returns over the period 01.03.2005 - 08.28.2015 comparatively in two parts; before 2010 when gold prices rose too high and after 2010. The results of the study concluded that the returns for 2007 and 2014 did not comply with Benford's Law. Similarly, Akkaş (2015b), in which the manipulation of foreign exchange rates was investigated regarding Benford's Law, tried to determine whether the digits patterns of USD/TL and EUR/TL exchange rates over the period 01.01.2003 -12.31.2014 complied with the second-order distribution of Benford's Law. As a result of the study, it was concluded that the digits patterns of USD / TL in 2003, 2004, 2008, 2010 and 2014, and of EURO / TL in 2005, 2006, 2007, 2009 and 2012 complied with the second-order distribution of Benford's Law.

In the study conducted by Uzuner (2017) investigating the manipulation in the banks traded in the stock exchange, the conformity of the first quarter consolidated balance sheets of the banks traded in the BIST throughout the first quarter of 2013 was examined. It was determined that the majority of the distribution of the first digits in the amounts of the accounts in the balance sheets of the relevant banks, whereas some of the accounts were different upon abiding by Benford's Law.

Chan, Liu and Xue (2013), in which the issue of manipulation in a limited number of cryptocurrencies in the literature was examined, investigated whether or not Bitcoin transaction data over the period from 11.26.2013 at 11:26:59 PM to 11.30.2013 at 09:42:41 AM comply with the first- and second-order distribution of Benford's Law. It was concluded that the deposit and withdrawal amount in general comply with the expected frequency distributions of Benford's Law. In a similar study investigating Bitcoin theft and fraudulent Bitcoin operations, Cipp, Phillips, and Mai (2013) investigated the compliance with Benford's Law by using all Bitcoin operations as a dataset over the period from the beginning of the network to 04.07.2013. According to the results of the study, it was determined that Bitcoin transaction data fitted very closely to Benford's Law frequency distribution.

In Wist (2019), the comformity of Bitcoin price and volume data obtained over the period between 04.28.2013 - 02.11.2018 with Benford's Law was tested. The results of the study indicated that the data closely comformed with Benford's Law.

In Peterson's (2020) study, all of the daily closing prices of Bitcoin over the period between July 2010 - May 2020 were analyzed using Benford's Law, with annual periods. It was concluded that Bitcoin was manipulated at 95% confidence intervals in 2013, at 95% in 2018, and at 98% in 2019.

Cong et al. (2019), in which the manipulation in different cryptocurrencies was investigated, examined the compliance of Bitcoin, Etherum, Ripple and Litecoin transaction data with Benford's Law. Transaction data obtained from 29 major cryptocurrency exchanges over the period between 09 July 2019 - 03 November 2019 were utilized in the analysis. According

to the results of the study, it was determined that the trading volumes of those cryptocurrencies were in line with Benford's Law for regulated exchanges. On the unregulated exchanges, it was determined that approximately 30% of the trading volume did not comply with Benford's Law.

In Veres' (2019) study, in which manipulation in a different cryptocurrency, Etherum, was investigated, analyzed all transactional data obtained from the Ethereum network until December 21, 2018 using Benford's Law. It was determined that the digits distribution of Etherum's transactional data did not comply with Benford's Law.

Amiram, Lyandres and Rabetti (2021) investigated the manipulation in Bitcoin, Etherum and Tether transactions of four different stock exchanges. Accordingly, the transaction data of these cryptocurrencies on Binance, Okex, ZB and Bibox stock exchanges over the period between June 2013 - September 2019 were compared with the Benford's Law reference distribution. It was determined that the transaction data complied with Benford's Law.

In studies focusing on the issue of manipulation in Bitcoin, the subject is explicated with Bitcoin transaction volumes. In this study, it is examined whether or not any manipulation exists using "daily returns" of Bitcoin, unlike other studies.

3. DATA AND METHODOLOGY

In the study conducted to determine whether or not any manipulation exists in the Bitcoin returns, the daily Bitcoin returns over the period from 02.02.2012 to 10.02.2020 constitute the dataset of the study. The data are obtained from BTC/USD daily returns found on the website www.investing.com. Daily returns are calculated using the formula $\frac{t_n - t_{(n-1)}}{t_{(n-1)}}$ as the rate of

return between two consecutive days. For the data to be suitable for the analysis, the rates of return are multiplied by 1000. Since there were no numbers that begin with zero in the analysis, 160 data are excluded from the analysis. Therefore, the number of observations is 3000.

The fact that the study is conducted merely on the Bitcoin's daily returns excluding the period prior to the year 2012 and other digital payment instruments via merely a single analysis method constitutes the constraints of the study.

The seminal studies on Benford's Law were conducted by Simon Newcomb (1881) and Frank Benford (1938). Newcomb (1881) explained his observations and conclusions about the frequencies of the use of digits. Newcomb realized that the first pages of the logarithmic chart used in the calculations were more worn-out than the last pages. Newcomb found that the probability of occurrence of the numbers that began with the digit "1" were higher than that of the numbers that began with the digit "2", whereas the probability of occurrence of the numbers that began with "3" in the logarithmic table. Similarly, he asserted that the smaller digits are used more frequently than, the larger digits. Newcomb provided the basis of the subject mathematically. After Newcomb, as of 1938, a physicist named Frank Benford in General Electric also found more worn pages than others in logarithmic tables. Benford (1938) studied the frequency distributions of 20,229 samples from different databases and calculated the mean values of these databases. The probability of occurrence of the digits in places has been entered into the literature as Benford's Law. Benford's Law is based on calculations of the probability of occurrence of the digits in the places of a number. The probability of occurrence of the digits in the first places of numbers is calculated as follows (Benford 1938, p. 554);

For the first digit of the numbers;

Probability
$$(d_1) = \log (1 + \frac{1}{d_1}); d_1 = (1, 2, 3 \dots 9)$$
 (1)

Following is a list of numbers for each place value indicating the probability ofoccurrence in a number according to the calculation above:

For the probability of 1; log $\left(1 + \frac{1}{2}\right) = 0.30103$	(2)
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For the probability of 2;
$$\log (1 + \frac{1}{2}) = 0.17609$$
 (3)

For the probability of 3;
$$\log (1 + \frac{1}{3}) = 0.12493$$
 (4)

For the probability of 4;
$$\log (1 + \frac{2}{4}) = 0.09691$$
 (5)

For the probability of 5;
$$\log (1 + \frac{1}{5}) = 0.07989$$
 (6)

For the probability of 6; $\log (1 + \frac{1}{6}) = 0.06695$ (7)

(8)

(10)

For the probability of 7; $\log (1 + \frac{1}{7}) = 0.05799$

For the probability of 8; $\log (1 + \frac{1}{2}) = 0.05115$ (9)

For the probability of 9; $\log (1 + \frac{1}{\alpha}) = 0.04576$

For the second digit of the numbers:

$$Probability(d_2) = \log \left(1 + (d_1d_2) - \log \left((d_1d_2); d_2 = (1,2,3...9\right)\right)$$
(11)

With the help of Benford's Law, the probability of distribution of digits found in all places can be analyzed. According to the Law, the probabilities of distribution of numbers tend to converge upon moving from left to right in the places.

Chi-square test is used to determine whether or not the distribution of daily Bitcoin returns complies with Benford's Law. Chi-square test is used to test whether or not the random probability distributions converge to a presumed theoretical distribution (Pearson 1900, s.157-175). In the Chi-Square test, the procedure applied to calculate the difference between observed and expected frequencies is as follows.

$$x^{2} = \sum_{i=1}^{9} \frac{(Pg, i - Pb, i)^{2}}{Pb, i}$$
(12)

P g,i: observed frequency for i digit

- P b,i: expected frequency for i digit
- $H_0\!\!:$ Observed values have similarities with the Benford Law's reference distribution.
- H₁: Observed values do not have similarities with the Benford Law's reference distribution.

For the Chi-Square test; the degree of freedom is v = r - 1 = 9 - 1 = 8. Since the significance level is 0.05, the critical value of the Chi-Square test is calculated as 15.505. Accordingly, if the Chi-Square test statistic exceeds 15.505, then the null hypothesis (H_o) can be rejected.

Bitcoin prices are determined by the equilibrium of supply and demand in the market. Nevertheless, the new Bitcoin supply can be realized by using mathematical algorithms in predetermined amounts at the system's inception. Unless generated Bitcoins are included in the market circulation, they are unable to influence the Bitcoin price. The volatility of Bitcoin prices and returns depend on whether or not natural and unnatural processes are included in the system. Excessive Bitcoin sales would influence the Bitcoin price and returns, even though there is no Bitcoin sale or production due to an excessive increase in production quantity. This can lead to manipulation of the Bitcoin price and returns. According to this approach, possible manipulations can be examined with Benford's Law. At this point, incompliance of the frequency of occurrence of the digits in Bitcoin returns with Benford's Law may reveal differences in digits data, deviation from the natural distribution of data and existence of manipulation.

4. FINDINGS

Within the context of the study, to detect any possibility of manipulation in Bitcoin returns, the Chi-Square test is performed to determine whether or not the distribution of Bitcoin returns calculated with daily Bitcoin price data obtained over the period of 02.02.2012 - 10.02.2020 comply with Benford's Law. Prior to performing the Chi-square analysis, the comparison between the distribution rates of the observed data of the Bitcoin returns used in the analysis and the reference distribution of Benford's Law is shown in Figure 1.

Upon examining Figure 1, it is seen that the observed data of daily Bitcoin returns corresponds to the digits probabilities of Benford's Law in 9 digits, and there is no significant discrepancy. It can be stated that digits 1, 3, 4, 6, 7, 8 and 9 are slightly below or above the required value, whereas the other digits 2 and 5 are almost exactly compatible. However, the measurement of the compliance of the observed and expected values; in other words, whether or not the deviations are within an acceptable range can be determined by the result obtained from the Chi-Square test.



Figure 1: Observed Values of Bitcoin Returns and Benford Probabilities

The results of the Chi-square test to determine the compliance of the observed data of daily Bitcoin returns with the reference distribution of Benford's Law during the period of 02.02.2012 - 10.02.2020 are presented in Table 1.

First Digits	Observed Values	Frequency	Expected Values	Benford Prob.	Proportional Diff.	Chi-Square
1	925	0.308	903	0.301	0.007	0.536
2	530	0.177	528	0.176	0.001	0.008
3	364	0.121	375	0.125	0.004	0.323
4	299	0.100	291	0.097	0.003	0.220
5	238	0.079	237	0.079	0.000	0.004
6	176	0.059	201	0.067	0.008	3.109
7	179	0.060	174	0.058	0.002	0.144
8	161	0.054	153	0.051	0.003	0.418
9	129	0.043	138	0.046	0.003	0.587
TOTAL	3000	1	3000	1	0.031	5.349

Table 1: Chi-Square Test Results

*Chi-Square Test Critical Value: 15.505

The Chi-Square distributions were calculated as 5.349 in the compliance test. The value obtained from the analysis is smaller than the critical value of the Chi-Square test (15.505). According to Chi-square test results, it is concluded that the difference between the frequencies of Bitcoin returns and the theoretical possibilities of Benford's Law would be assumed to be random at the accepted significance level.

Furthermore, in accordance with the findings obtained from the analysis, the absolute proportional difference between the observed data and of the Law is 0.031 on average. In order to calculate the mean of this difference which constitutes the general sum, it is necessary to divide the value by 9. According to the above data, the observed rate deviates from the Benford ratio (0.031/9=0.0034) by 3.4 per thousand. However, it is considered that approximately 3.4 per thousand deviation in the database consisting of 3000 data is very low. The very low mean of deviation in these 9 digits indicates that the data abides by Benford's Law at very close frequencies. As a result of the analysis, compliance of the frequency of occurrence of the digits in Bitcoin returns with Benford's Law, absence of any difference on the digits data, and inexistence of deviation from the natural distribution of the data connote the absence of any external intervention. Therefore, during the period from 02.02.2012 to 10.02.2020, it is considered that there is no manipulation in daily Bitcoin returns.

5. CONCLUSION

Due to the cryptocurrencies' rapid entry into the financial system which connotes a different payment/investment instrument; those currencies are approached prudently by individuals, institutions, and governments. There are many

individuals and organizations interested in the Bitcoin currency, which has an increasing trend in terms of usage and prevalence. Along with its increasing popularity, there is an increasing number of question marks in minds regarding whether or not Bitcoin is not priced correctly; in other words, any manipulation of Bitcoin returns exists.

There is a limited number of empirical studies in the literature conducted on manipulation in Bitcoin and other cryptocurrencies, which highly attract attention in the financial system. The issue of manipulation in Bitcoin are examined with Bitcoin transaction volumes. Unlike other studies, it is examined whether or not manipulation exists by using Bitcoin returns in this study. This situation increases the importance of the research study.

In order to determine whether or nor any manipulation exists in Bitcoin returns, the Bitcoin data are analyzed using the daily Bitcoin data throughout the period from 02.02.2012 to 10.02.2020, and Chi-Square test is performed to detect whether or not the distribution of Bitcoin returns complies with Benford's Law. According to the results of the analysis, the hypothesis suggesting that the Bitcoin returns are the same as the Benford Law's reference distribution is accepted. Therefore, it is concluded that there is no manipulation in the Bitcoin returns exhibit natural distribution and there is no manipulation. The results obtained from the analysis, compared to Chan, Liu, and Xue (2013), Cipp, Phillips, and Mai (2013), Wist (2019) and Cong et al. (2019) which investigated possible manipulations using Bitcoin transaction data, can be said to have reached similar outcomes in general.

Upon considering the large fluctuations in Bitcoin prices, it is crucial for investors and stock exchanges to make sure that there are no fraudulent transactions. The fact that the prices are formed according to the natural supply-demand balance would not change the fact that Bitcoin is an investment tool with high volatility. In this context, investors can only be recommended to prefer Bitcoin investment following the risk and return optimization processes.

The application of Benford's Law in the analysis can provide significant contributions such as cost and time savings. Nonetheless, it is thought that it would be more fruitful to use more than one audit technique concurrently rather than merely utilizing Benford's Law. This can be the subject of future studies.

REFERENCES

Akkas, M. E. 2015(a). Altın getirileri dağılımının Newcomb-Benford kanunu ile tespiti. Uluslararası Sosyal Araştırmalar Dergisi. 8(40), 577-584.

Akkas, M. E. 2015(b). ABD doları ve Avro döviz kurlarının nümerik desen analizi. Karabük Üniversitesi Sosyal Bilimler Enstitüsü Dergisi. 5(2), 215-226.

Alagoz, A., Ay, M. (2001). Muhasebe denetiminde Benford Kanunu temelli dijital analiz. Selçuk Üniversitesi Sosyal ve Ekonomik Araştırmalar Dergisi.(4), 59-76.

Amiram, D, Lyandres, E., Rabetti, D. (2021). Competition and product quality: fake trading on crypto exchanges. Available at SSRN: https://ssrn.com/abstract=3745617 or http://dx.doi.org/10.2139/ssrn.3745617.

Benford, F., (1938). The law of anomalous numbers. American Philosophical Society. 78(4), 551-572.

Chan, J., Liu, T., Xue, E., (2013). Analyzing the Bitcoin transaction graph: a look at mixers and traceability. https://css.csail.mit.edu/6.858/2013/projects/jeffchan-exue-tanyaliu.pdf

Cipp, C., Phillips J., Mai, A, (2013). The hacker's code: finding Bitcoin thieves through the similarity and status claims between users. Stanford CS 224W - Team 39C – S 224W Final Report.

Cong, L. Li, X. Tang, K., Yang, Y. (2019). Crypto wash trading. Available at SSRN: https://ssrn.com/abstract=3530220 or http://dx.doi.org/10.2139/ssrn.3530220.

Carkacıoglu, A., (2016). Kripto-para Bitcoin. Sermaye Piyasası Kurulu Araştırma Raporu. Ankara.

Dodgson, M., Gann, D. M., George, G., (2015). Managing digital money. The Academy of Management Journal.58 (2), 325-333.

Doğan, Z., Buyrukoglu, S. and Kutbay, H. (2018). Türkiye'de Bitcoin işlemlerinin vergilendirilmesi ve muhasebeleştirilmesine ilişkin oneriler. *Vergi Sorunları Dergisi* (361), 32-40.

Houben, R., Snyers, A. (2018). Cryptocurrencies and blockchain legal context and implications for financial crime, money laundering and tax evasion. The European Parliament's Special Committee on Financial Crimes.

http://www.europarl.europa.eu/cmsdata/150761/TAX3%20Study%20on%20cryptocurrencies%20and%20blockchain.pdf,

Jermann, U. J. (2018). Bitcoin, Cagan's model of hyperinflation. Available at SSRN:https://ssrn.com/abstract=3132050 or http://dx.doi.org/10.2139/ssrn.3132050

Gandal, N., Hamrick, JT, Moore, T., Oberman, T. (2017). Price manipulation in the Bitcoin ecosystem. *Journal of Monetary Economics*. DOI:10.1016 https://tylermoore.utulsa.edu/jme17appendix.pdf

Ley, E. (1996). On the peculiar distribution of the U.S. stock indexes digits. The American Statistician.50(4), 311-313.

Newcomb, S. (1881). Note on the frequency of the use of digits in natural numbers. American Journal of Mathematics.(4), 39-40.

Pearson, K.(1900). On a criterion that a given system of deviations from the probable in the case of a correlated system of variables is such that it can be reasonably supposed to have arisen from random sampling. *Philosophical Magazine*. 5(50), 157–175.

Poyser, O. (2017). Exploring the determinants of Bitcoin's price: an application of bayesian structural time series. Dissertation. https://arxiv.org/ftp/arxiv/papers/1706/1706.01437.pdf

Peterson, T. (2020). To the moon: a history of Bitcoin price manipulation. *Journal of Forensic and Investigative Accounting*. 13(2), Available at SSRN: <u>https://ssrn.com/abstract=3639431</u> or <u>http://dx.doi.org/10.2139/ssrn.3639431</u>

Sermaye Piyasası Kurulu (2003). Hisse senedi piyasasında manipülasyon: kullanılan yöntem örnekleri, manipülatif işlem kalıbı örnekleri, korunma yolları. http://www.spk.gov.tr/Duyuru/Dosya/20031215/0

Shengmin, Z., Wenchao, W. (2010). Does Chinese Stock Indices agree with Benford's Law?. *Management and Service Science (MAASS)*. 2010 International Conference on Management and Service Science. DOI:10.1109/ICMSS.2010.5575999

Uzuner, M.T.(2014). Benford Yasasının Borsa İstanbul'da işlem gören bankaların konsolide bilançolarına uygulanması. Finansal Araştırmalar ve Çalışmalar Dergisi. 5(10), 73-82.

Veres, I. (2019). Identification of unusual transactions in Blockchain networks. Master's Thesis. Slovak University of Technology in Bratislava. Faculty of Informatics and Information Technologies. Institute of Computer Engineering and Applied Informatics.

Wist, H. (2019). Assessing the conformity of cryptocurrency market data with Benford's Law. Master's Thesis, University of Vaasa. Faculty of Business Studies. Department of Accounting and Finance.